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ON THE AMOUNT OF INTERNAL FRICTION DEVELOPED IN ROCKS DURING DEFORMATION AND ON THE RELATIVE PLASTICITY OF DIFFERENT TYPES OF ROCKS

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INTRODUCTION

At the meeting of the Geological Society of America held in Albany in the year 1900, a brief résumé of the experimental work on the flow of marble carried out by Adams and Nicolson was presented to the Society, and in the discussion which followed the reading of this paper a number of interesting points were suggested by various speakers as worthy of experimental investigation. Among these was one put forward by Dr. G. K. Gilbert, which, in a letter to the authors, he subsequently formulated as follows:

It has been thought that great pressure breaks down the structure called solidity and so reduces viscosity that very little differential stress is necessary to produce flow. It is thought that the strength of rocks is practically unaffected by pressure, in which case flow should begin only when differential stress equals the crushing strength of the material as conditioned by the temperature. It is certainly conceivable also that the strength of rocks is increased by pressure, so that the production of flow requires differential stress greater than the crushing stress as conditioned by the temperature. I hope your experimentation may be brought to throw light upon this point.

The sense in which certain terms are used in this quotation is not quite clear, but we understand the question put forward by Dr. Gilbert to be as follows:

A unit cube of any rock—granite for instance—is submitted to pressure in a testing machine on the earth's surface. It will give away or break down under a certain load—this is termed its crushing load.

If this cube of rock were imbedded deep within the earth's crust, great pressure would be exerted upon it from all sides. Such being the case, and omitting from consideration the influence of temperature, would the rock (1) be reduced to a condition which approaches fluidity and move at once if the pressure in one direction became slightly greater than that in another? Or (2) would the rock become deformed only when this additional pressure in one direction was equal to its crushing load at the surface? Or (3) would the rock show an increased resistance to deformation and require a much greater additional pressure in one direction to deform it than was required to crush it at the surface?

A few preliminary trials which served to open up the experimental investigation of this problem were undertaken some years ago by Dr. Adams in association with Dr. Ernest G. Coker, then Associate Professor of Civil Engineering at McGill University. Dr. Coker subsequently resigned his position at McGill University to accept the professorship of mechanical engineering and applied mathematics at the Finsbury Technical College in London, and for a time the work was discontinued. Dr. Bancroft, however, some years later coming to McGill University, the investigation was resumed. It has extended over a period of several years. The writers desire to acknowledge their indebtedness to the Carnegie Institute of Washington, the work having been carried out under a grant received from that body.

ROCKS EXAMINED

The following rocks were examined:

White alabaster, Castelino, Italy.

White marble, Carrara, Italy.

Black Belgian marble ("Noir fin").

White dolomite, Cockeysville, Maryland, U.S.A.
Steatite ("Albarine"), Virginia, U.S.A.
Slate, New Rockland, Province of Quebec, Canada.
Sandstone, Cleveland, Ohio, U.S.A.
Granite, Baveno, Italy.
Olivine diabase, Sudbury, Province of Ontario, Canada.

For the purposes of comparison experiments were also conducted with metallic copper and metallic lead.

Detailed petrographical descriptions of these rocks, with the exception of the alabaster, dolomite, steatite, and slate, have been given in a former paper.[1] It is necessary here, therefore, to refer briefly to the character of these four rocks only.

Alabaster, Castelino, Italy.—Under the microscope the rock is seen to be composed of an aggregate of small grains of gypsum which are clear, colorless, and approximately equal in size. The individual grains display a tendency to elongation in one direction, thus giving the rock a very faint foliation. The columns of alabaster used in the experiments were cut from a single uniform block of this rock in such a manner that their longer axes were parallel to this indistinct foliation.

Dolomite, Cockeysville, Maryland, U.S.A.—This is a rather fine-grained, white, granular dolomite, very pure in character and uniform in composition, containing $CaCO_3$ and $MgCO_3$ in almost exactly their molecular proportions. It presents the appearance of a white marble and is extensively quarried as such. Thin sections of the rock, when examined under the microscope, show that it is composed of a mosaic of grains of the mineral dolomite, more or less irregular in shape and varying somewhat in size. Between crossed nicols, they present a uniform extinction or show only the faintest strain shadows. They are very seldom twinned.

Steatite, Virginia, U.S.A.—This steatite is placed on the market under the name of "albarine." The columns employed in the experiments were cut from a perfectly uniform slab of this rock

[1] "An Investigation into the Elastic Constants of Rocks More Especially with Reference to Their Cubic Compressibility," by F. D. Adams and E. G. Coker, The Carnegie Institute of Washington, 1906; see also *American Journal of Science*, XXII (August, 1906).

with dimensions of $10'' \times 11'' \times 1\frac{1}{4}''$. Under the microscope the rock is seen to possess a distinct foliation parallel to the broad surface of the slab. All of the columns were cut from this slab with their longer axes parallel to the foliation. In thin sections under the microscope the rock is seen to be composed chiefly of chlorite, talc and dolomite, numerous small crystals and grains of magnetite, and a few grains of pyrite are also present. The two minerals, chlorite and talc, make up by far the greater portion of the rock, the chlorite being somewhat more abundant than the talc. Both occur as plates and sheaflike aggregates, and both possess a very distinct cleavage parallel to which extinction takes place. The dolomite is present both in large rhombohedral individuals and as small irregular granules which possess a linear arrangement parallel to the foliation of the rock. None of the grains of dolomite show either twinning or strain shadows. Having been cut parallel to the foliation, it is not surprising that the columns of this rock employed in the experiments bulged assymetrically when deformed, and hence a larger number of experiments were made with the steatite than with the other rocks, in order that accurate average results might be secured.

Slate, New Rockland, Quebec, Canada.—This is a typical fine-grained slate, black in color, uniform in character, and possessing an excellent cleavage. By means of a diamond drill cores were taken perpendicular to the cleavage of the slate, and from these the columns of slate used in the experiments were prepared.

Under the microscope this slate is found to be composed essentially of minute flakes of two minerals, one of which is apparently kaolin and the other muscovite. In general, the kaolin is much more abundant than the muscovite, from which it can be distinguished in that it possesses a lower double refraction and is not quite so transparent. Within a few extremely narrow bands of the slate the muscovite preponderates. A few minute grains of quartz are interposed between the flakes of muscovite and kaolin. A considerable number of very small flakes of black, opaque, carbonaceous matter, abundant, minute, needle-like crystals of rutile, and a very few widely scattered grains of pyrrhotite are also present. The

rutile crystals are brownish in color and occasionally display the geniculated twinning that is characteristic of this species.

The foliation of the slate explains the lack of symmetry in the expansion of columns of this rock during deformation.

The *Copper* used in these experiments was taken from a rod 1 inch in diameter, representing a good commercial grade of this metal. Prior to being turned into columns for the experiments, the pieces cut from the rod were annealed by being heated to bright redness in the coal fire of a forge, being then allowed to cool down gradually.

The *Lead* employed in the experiments was "assay lead" which, in order to free it from all air bubbles, was melted down and cast in a heated iron mold, which was then allowed to cool slowly.

METHODS EMPLOYED

Several long round bars of nickel steel 2½ inches in diameter, all of identical composition and from the same heat, and all having been submitted to identical treatment in their manufacture, were secured. For these the authors are indebted to the Bethlehem Steel Company, which placed them at their disposal for the purpose of the present investigation.

This steel, which is very uniform in character, possesses a high tensile strength, as well as a high elastic limit, and has the following chemical composition:

Carbon	.30 per cent
Manganese	.74 per cent
Silicon	.162 per cent
Phosphorus	.035 per cent
Sulphur	.038 per cent
Nickel	4.740 per cent

The bars were sawed into lengths of about 3¼ inches. These were then bored and turned into tubes, the longitudinal sections of which, with the final dimensions, are shown in the upper half of Fig. 1. Two sets of these tubes were prepared, differing only in the thickness of the wall of the central portion of the tube. In the first set this has a thickness of 0.33 centimeter, while in the second

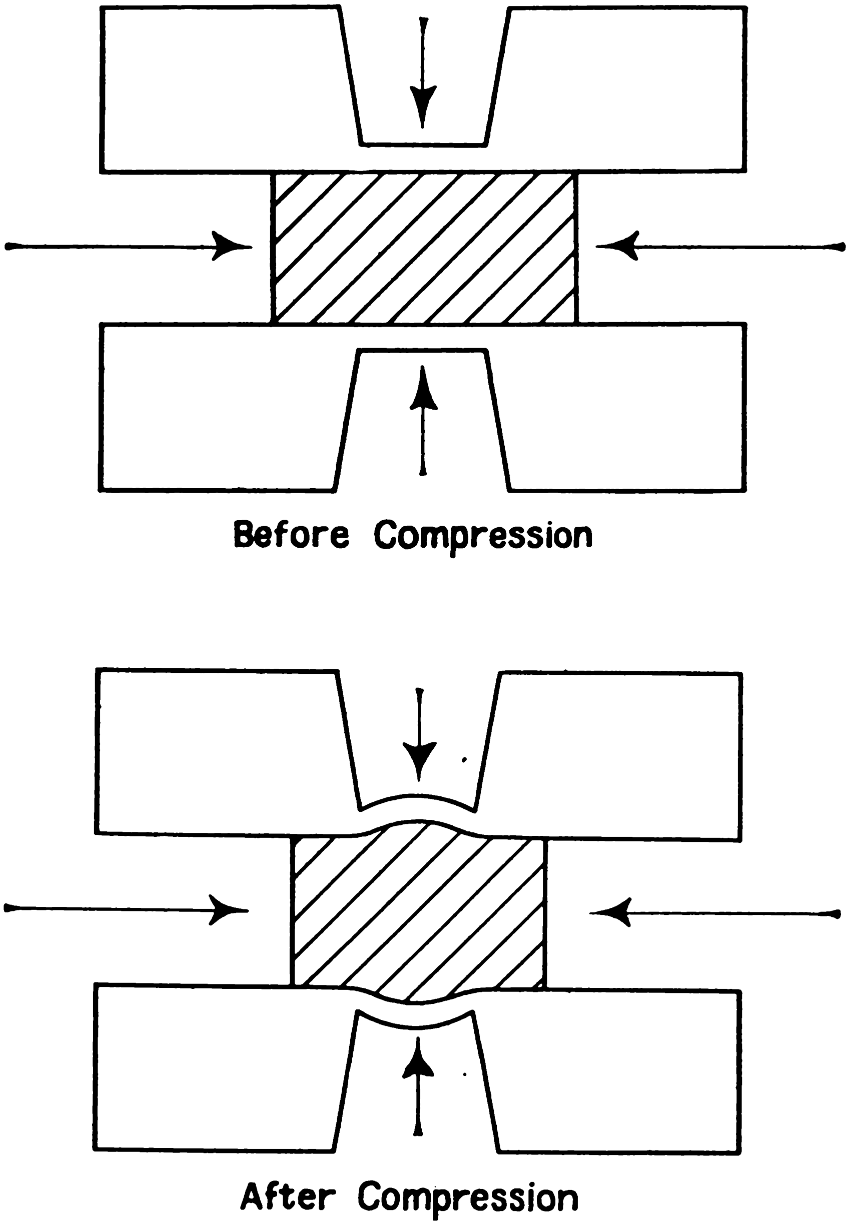

FIG. 1.—Longitudinal section through steel cylinder with wall 0.33 cm. thick, and inclosing one of the rock columns (natural scale).

set the thickness is 0.25 centimeter. The interior diameter of the tube in both sets is of such a size that it will just receive a column of rock 2 centimeters in diameter. The inner surface of the tube in every case was not only perfectly smooth, but highly polished. The angle of the bevel, by which the thickness of the wall is reduced at the middle of the tube, was adopted after a long series of preliminary experiments, which proved it to be that which was demanded by the conditions to be secured. Pistons fitting accurately into either end of these tubes were then made of chromium tungsten steel, suitably tempered by being heated, quenched in oil, and then ground to the exact dimensions required.

Large blocks of each of the rocks having been secured, rough columns of them were bored out by means of a hollow-bit diamond drill, care being taken in the case of each rock to have all the columns bored out of the rock in the same direction, that is, parallel to one another, so that any possible variations due to rift, grain, or incipient foliation were avoided. These rough columns were then reduced to the exact size required, by being ground down in a lathe by means of revolving carborundum wheels of different degrees of fineness, and were finally highly polished. When completed the columns were of such a size that they would just pass into the steel tubes at the ordinary temperature, the tube inclosing the column with an absolutely perfect mechanical fit. The column was in each case 4 centimeters long and 2 centimeters in diameter. While the column was thus fitted accurately into the tube, it could, by the exertion of a certain amount of pressure, be moved up and down within the tube. The column of rock, when inserted into the tube, was so placed that its center was exactly in the center of the thinner portion of the tube, as shown in the diagram, the extremities of the column being in this way supported by the walls of the thicker portion of the tube at either end.

The pressure to which the rock was submitted was obtained by a Wicksteed testing machine set up in the Testing Laboratory of the Macdonald Engineering Building of McGill University. This machine has a capacity of 100 tons and, when loaded to its capacity, is sensitive to a load of 4 pounds. Unfortunately, being graduated to read only in tons and pounds, it was necessary to obtain the data

of the research in these units. In presenting the final results, however, the data for the conversion of these into a unit more generally employed in physical investigations are given.

The extensometer employed for the purpose of measuring the expansion of the tube under pressure was a simplified form of the type designed by Professor Coker and described in the *Proceedings of the Royal Society of Edinburgh*, XXV (1904–5). It was affixed to the opposite points of the steel tube on the plane of maximum deformation and showed the expansion, multiplied by two, by means of a fine line moving over a graduated scale, which was read by a telescope placed at a distance of several feet.

In a number of experiments two extensometers were employed, which were applied to the tube in the plane of maximum deformation, but in directions at right angles to one another. In this way it was ascertained that the bulge which the steel tube displayed under pressure was nearly symmetrical, but in order that any error which might arise from a single measurement might be eliminated, in almost all cases the two extensometers employed were affixed to the tube at right angles to one another, and the mean of the two readings was secured. By means of this form of extensometer and by reading with a telescope, it was possible to measure an increase on the diameter of the tube amounting to only 0.0005 inch. The steel tube inclosing the rock column, with the extensometers in position, the whole set up in the press ready for the application of pressure, is shown in Fig. 2.

The method adopted for measuring the internal friction developed in the rock by deformation was as follows:

A column of rock, Carrara marble, first was taken, having the dimensions already referred to. This was inclosed in a tube of nickel steel, as above described; the tube had a wall thickness of 0.25 centimeter at its thinner portion. As will be seen from Fig. 1, the middle portion of the marble column is inclosed by the thinner portion of the tube, while the ends of the column are held by the thicker portion of the tube wall. In this way the rock is prevented from flowing up between the tube and the pistons and thus from escaping from the tube. With a tube of this shape and these dimensions, the movement of the rock under pressure is confined

to the middle portion of the column, which is surrounded by the thinner portion of the tube. The pistons being inserted and the whole properly set up in the testing machine, the pressure was

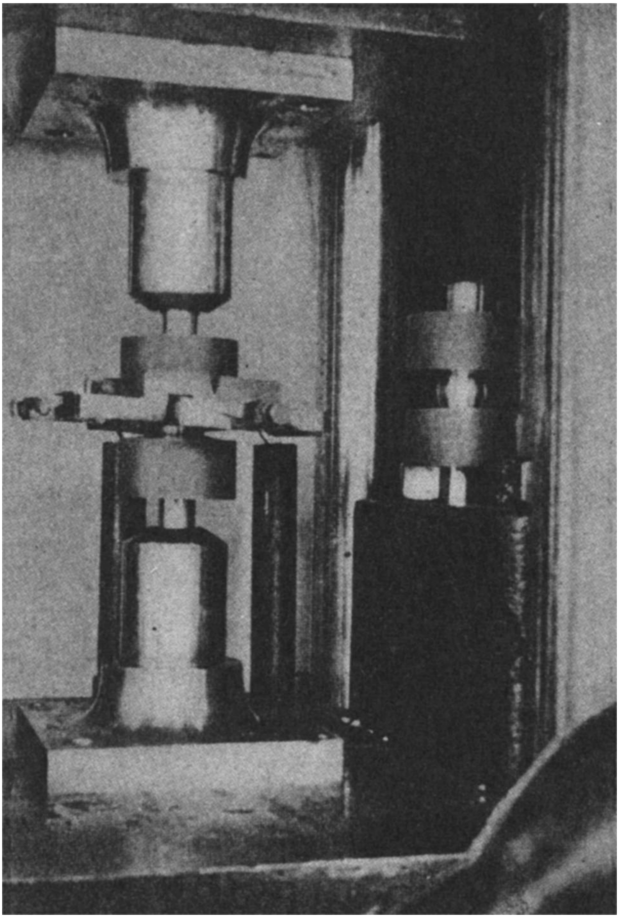

FIG. 2.—Steel cylinder, inclosing a rock column and with the two extensometers in position, set up in the Wicksteed press. To the right a bulged cylinder is shown as it appears at the close of an experiment.

applied in successive increments of 1,000 pounds. The extensometer showed no yielding of the inclosed rock until a load of about 12,000 pounds had been reached, when a very slight distension of

the tube was indicated. Up to this point, the marble, being an elastic body, was undergoing cubic compression, the pressure exerted by the machine and the resistance exerted by the steel collar being equal. The slight distension of the steel tube at a load of 12,000 pounds is due to the elastic deformation of the marble. After each additional increase of 1,000 pounds to the load, extensometer readings were taken every 30 seconds until four successive readings were identical, that is to say, until no movement that could be registered on the scale took place during a period of 2 minutes. The pressure was then increased by another 1,000 pounds and a similar series of readings were taken. This was continued until the bulging steel tube showed signs of rupture or was actually ruptured by the movement of the inclosed rock. The time which elapsed between the first application of pressure and the final rupture of the tube, that is to say, the duration of the experiment, differed somewhat in the different experiments, but may be said to be about four hours.

During the time which elapses from the point when the elastic limit of the rock is exceeded to that at which the tube fails, the inclosed rock is undergoing deformation with extreme slowness and by internal movements of one kind or another, which give rise to what may be termed a plastic flow.

At the commencement of the experiment the column of marble had the form and dimensions represented in the upper half of Fig. 1. When at the conclusion of the experiment the test piece was placed in a lathe and the steel collar was turned off, the specimen of marble was set free. It was still intact, unbroken, and, when tested in compression, was found to be very nearly as strong as a piece of the original marble of the same shape and size. It now had the form represented in the lower half of Fig. 1.

A photograph of a column of rock, before and after deformation, the rock, however, in this particular case being steatite, is shown in Fig. 3.

The pressure which was applied to the marble column effected two results. It overcame the pressure (or resistance) exerted upon the sides of the column by the inclosing tube of steel, and it overcame the internal friction developed within the rock during its

change of shape. If it were possible, therefore, to ascertain the amount of the pressure (or lateral resistance) exerted by the inclosing tube, it would be possible by subtracting this from the total load employed to determine the load which was required to overcome the internal friction of the rock under the conditions of the experiment.

In order to determine the amount of pressure required to effect the progressive deformation of the tube, i.e., the amount of pressure exerted by the tube on the inclosed rock during the successive stages of deformation, a series of steel tubes, identical in every respect with those employed in the experiment just described, were taken and were deformed in a precisely similar manner, except that these tubes were filled with soft tallow, instead of being occupied by a column of marble. This material was selected as being one which moves with the development of an amount of internal friction which is so small that it was negligible in the present case. In carrying out the experiment with tallow, we found it necessary to slightly alter the shape of the steel pistons, the ends inserted in the steel tube being turned so as to present a somewhat concave face, as shown in Fig. 4, the outer margins having a thin feather edge. When pressure is brought to bear upon these pistons, this thin edge expands slightly, thus pressing against the walls of the tube and preventing the tallow from escaping between the piston and the wall. It was found that in this way the deformation of the tube could be readily effected.

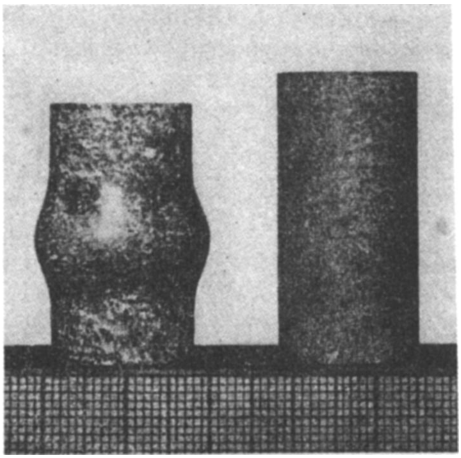

FIG. 3.—Photograph of columns of steatite before and after deformation. The smaller divisions of the scale below are millimeters.

The objection might be put forward that, while undoubtedly the tallow possesses at ordinary atmospheric pressure an internal friction which is quite negligible, this material under the pressure to which it must be subjected in order to deform the steel tube might develop an amount of internal friction and a rigidity which would be by no means negligible.

In order to ascertain whether such was the case, companion experiments were made, using the same pistons, but employing

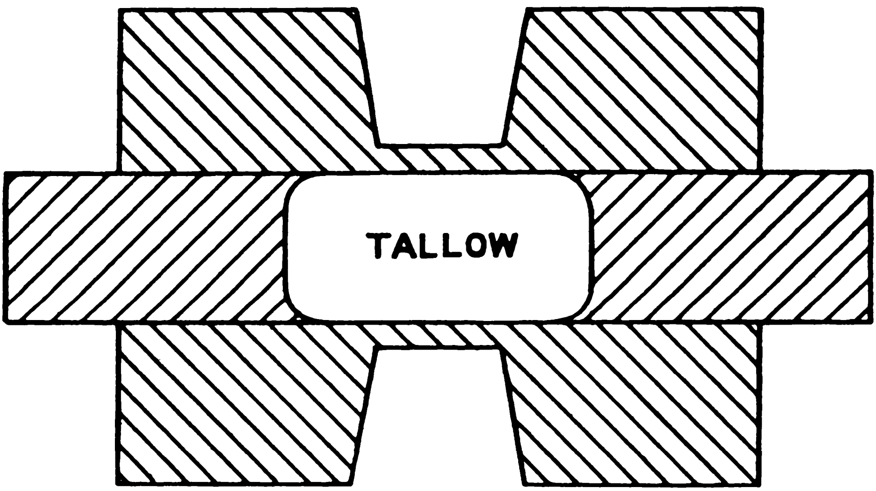

FIG. 4.—Longitudinal section through steel cylinder, showing the type of piston used when deforming the steel with tallow.

water in one case and oil in another, instead of tallow. It was found that the deformation of the tube could be effected by either of these materials, although, when water was employed, it was necessary to raise the pressure rapidly at first to cause the feather edges of the pistons to expand and make the joint tight, thus preventing the water from escaping. This series of comparative experiments was carried out with loads up to 19,000 pounds, at which pressure the tubes failed, and it was found that under these pressures the three substances mentioned—water, oil and soft tallow—showed no difference in viscosity which could be detected. The tallow, of course, undoubtedly possesses a somewhat greater inter-

nal friction than the water, but at the range of pressure to which it was submitted in the present investigation this difference is not noticeable and may therefore be neglected. The tallow, however, being more convenient for purposes of experiment, was employed in a further series of comparative experiments.

There was one other possible source of error, namely, the friction between the walls of the tube and the thin feather edge of the hollow-faced piston used in the experiments with the tallow. In the experiments with a column of rock a flat-faced piston was of course employed, and this source of friction was thus eliminated. In order to ascertain the amount of this friction in the case of the tallow, another steel tube was constructed, identical in all respects with those used in this investigation. One end of it, however, was closed so that it would be necessary to employ only a single piston, and through the closed end a small copper tube was inserted, which led to a powerful pump provided with an accurate pressure gage. The whole apparatus having been filled with water supplied by the pump, the steel tube with its cup-shaped piston was placed in a 75-ton Emery testing machine, and the piston slowly forced into the fluid, the pressure required to do this being noted at every stage on the testing machine and also on the gage fitted to the pump. In this way the pressure necessary to force the piston forward was measured at each additional increment of load applied to the piston by the Emery machine. As a result of a series of trials, it was ascertained that the friction on the feather edges of the piston amounted on an average to only 290 pounds, so that, in view of the very heavy pressure employed in this investigation, the error thus introduced is so small that it may be neglected.

It having been ascertained that soft tallow was a material which for the purposes of this investigation might be considered to move without the development of internal friction, a series of experiments were made with steel tubes identical in character and dimensions with those employed to inclose the marble, but soft tallow was substituted for marble. The two series of experiments were carried out in exactly the same manner in every detail, except that in the tubes filled with tallow the load was raised by increments of 500 pounds, instead of 1000 pounds, and the readings were taken

every 15 seconds instead of every 30 seconds till they remained constant for at least 5 consequent readings. This change was necessitated in order to standardize the conditions in the two series of experiments, since, when the tube was filled with tallow, the whole load was applied to overcome the resistance of the tube, while, when the place of the tallow was taken by marble, a portion of the load was applied to overcome the internal friction of the rock, and the movement was slower. By modifying the procedure, as above mentioned, in the case of the tubes filled with tallow an identical deformation was secured in both cases.

When columns of rock are inclosed in the steel tubes and deformation is carried out in the manner described, the impending rupture of the steel tube, which marks the conclusion of the experiment, is indicated by the appearance of a series of sharply marked vertical lines on the bulged wall of steel which inclosed the deformed rock. If the experiment is continued, the tube splits along one of these vertical lines, and the inclosed rock becomes visible, and, if the pressure is still maintained, the resistance along the line of rupture being removed, the rock along this line crumbles and is forced out of the fissure in the form of a powder.

In the case of the experiments in which tallow was employed in place of a column of rock, the completion of the test is marked by the development of a vertical fissure in the thin portion of the steel tube in the usual manner. So soon as this appears, however, and usually before the load can be taken off the testing machine, a fragment of the thin steel wall, bounded on one side by the fissure in question and at the top and bottom by the thicker portion of the steel tube, opens out like a door on its hinges and is instantly torn off and with a loud report is shot across the room with great violence. It therefore was necessary in the case of these experiments that the observer should always be protected from these projectiles, the importance of this protection being emphasized in the case of one of the experiments by the fact that the piece of steel struck and split in two the piece of hard wood, a quarter of an inch thick, which protected the observer's head.

In order to make quite sure that the form and outline of the bulge assumed by the tube in the case of the experiments with the

different rocks was the same, a special series of experiments to decide this question was made, employing copper, lead, marble, Belgian black, and granite. In each instance the experiment was carried to the point where the bulge or expansion of the diameter amounted to 0.030. The cylinder was then removed, and by using an electric arc light in a dark room a sharp shadow of the outline of the bulged cylinder was cast upon sensitive paper, removed at such a distance that the photograph enlarged the outline of the cylinder approximately 18 times. The cylinder was then placed in the Wicksteed machine, and the bulge increased to 0.110, and a similar photograph taken. By a comparison of the photographs it was found that the outline of the deformed wall was essentially identical in all cases.

As has been mentioned, from two to five experiments were made in the case of each rock when inclosed in a 0.25-centimeter tube and the same number with each rock inclosed in a tube having a wall thickness of 0.33 centimeter. The mean of the closely concordant results was then worked out in each case, and the figures obtained are presented in Tables I and II. These represent the data yielded by the experimental work.

The necessary data having been thus secured, a curve was plotted presenting these graphically in the case of each experiment. In these curves the exact amount of the load required to produce any required bulge or distension of the tube is shown from the point when the first movement can be detected until the final rupture of the tube takes place. The curves for the several experiments with Carrara marble inclosed in the steel tubes with a 0.25-centimeter wall are shown in Fig. 5 (p. 620). A curve representing the mean of the results obtained in the several experiments is also given. In Fig. 6 (p. 621) this curve of the mean of the results obtained from the marble inclosed in a 0.25-centimeter tube is reproduced, and below it is the mean of the curves obtained from tallow when inclosed in a 0.25-centimeter steel tube.

Since the tallow, as has been shown, offers itself no measurable resistance to deformation under the conditions of the experiment, the curve in the tallow experiments shows merely the resistance offered to deformation by the steel tube itself.

TABLE I

AMOUNT OF DEFORMATION, IN INCHES, OF THE SEVERAL ROCKS WHEN INCLOSED IN THE STEEL CYLINDERS HAVING A WALL THICKNESS OF 0.25 CENTIMETER

Load in Pounds*	Tallow	Lead	Copper	Rocks Employed								
				Steatite, Virginia, U.S.A.	Alabaster, Castelino, Italy	Sandstone, Cleveland, Ohio, U.S.A.	White Marble, Carrara, Italy	Dolomite, Cockeysville, Maryland, U.S.A.	Black Belgian Marble ("Noir fin")	Slate, New Rockland, Quebec, Canada	Olivine Diabase, Sudbury, Ontario, Canada	Granite, Baveno, Italy
2,000	0.0002	....	....	....	....	....	....	....	....	....	....	....
3,000	.0005	....	....	....	....	....	....	....	....	....	....	....
4,000	.0008	....	....	....	0.0003	....	....	....	....	....	....	....
5,000	.0013	0.0004	....	....	.0003	....	....	....	....	....	....	....
6,000	.0018	.0010	....	....	.0003	....	....	....	....	....	....	....
7,000	.0025	.0015	....	0.0003	.0003	....	....	....	....	....	....	....
8,000	.0032	.0020	0.0003	.0004	.0003	....	....	....	....	....	....	....
9,000	.0062	.0038	.0005	.0005	.0003	....	....	....	....	....	....	....
10,000	.0130	.0075	.0010	.0006	.0003	....	0.0003	....	....	....	....	....
11,000	.0216	.0139	.0018	.0006	.0008	....	.0005	....	....	....	....	....
12,000	.0323	.0220	.0023	.0008	.0009	....	.0006	....	....	....	....	....
13,000	.0483	.0330	.0038	.0009	.0009	....	.0006	....	....	....	....	....
14,000	.0748	.0550	.0050	.0009	.0010	....	.0008	....	....	....	....	....
15,000	0.1097	.0843	.0073	.0013	.0013	....	.0009	....	....	....	....	....
16,000	....	0.1133	.0098	.0013	.0013	0.0003	.0009	....	....	....	....	....
17,000	....	....	.0120	.0016	.0014	.0005	.0016	....	....	....	....	....
18,000	....	....	.0148	.0021	.0016	.0005	.0018	....	....	....	....	....
19,000	....	....	.0175	.0026	.0021	.0005	.0018	....	....	....	....	....
20,000	....	....	.0203	.0043	.0028	.0008	.0021	....	....	....	....	....
21,000	....	....	.0233	.0068	.0031	.0008	.0021	....	....	....	....	....
22,000	....	....	.0260	.0103	.0036	.0010	.0023	....	....	....	....	....

23,000			.0295	.0145	.0045	.0010	.0028					
24,000			.0328	.0199	.0063	.0015	.0030					
25,000			.0365	.0258	.0080	.0023	.0035					
26,000			.0403	.0323	.0098	.0030	.0043				0.0003	
27,000			.0445	.0386	.0126	.0040	.0051				.0003	
28,000			.0488	.0462	.0151	.0065	.0068				.0004	
29,000			.0535	.0532	.0184	.0083	.0078	0.0005			.0004	
30,000			.0583	.0608	.0218	.0125	.0085	.0009	0.0005		.0004	
31,000			.0640	.0694	.0258	.0155	.0110	.0011	.0010		.0006	
32,000			.0708	.0779	.0318	.0198	.0126	.0011	.0010		.0006	0.0004
33,000			.0768	.0872	.0353	.0238	.0139	.0011	.0010		.0009	.0004
34,000			.0840	.0972	.0419	.0283	.0169	.0013	.0010		.0009	.0004
35,000			.0923	.1072	.0473	.0330	.0181	.0016	.0010		.0009	.0004
36,000			.0995	.1184	.0556	.0368	.0208	.0031	.0013		.0009	.0004
37,000			0.1093	.1300	.0868†	.0423	.0234	.0036	.0013		.0014	.0004
38,000				0.1512	.0893	.0483	.0268	.0040	.0018		.0014	.0004
39,000					.0935	.0535	.0296	.0040	.0020		.0014	.0004
40,000					.0990	.0593	.0326	.0045	.0020	0.0004	.0014	.0004
41,000					.1048†	.0650	.0354	.0048	.0020	.0008	.0014	.0004
42,000					.1123	.0738	.0386	.0053	.0023	.0008	.0018	.0004
43,000					.1231	.0800	.0416	.0060	.0023	.0009	.0018	.0008
44,000					.1324	.0880	.0464	.0075	.0025	.0009	.0018	.0008
45,000					.1458	.0958	.0511	.0091	.0030	.0009	.0020	.0010
46,000					0.1559	.1050	.0560	.0108	.0033	.0010	.0020	.0010
47,000						.1145	.0613	.0129	.0038	.0011	.0025	.0010
48,000						.1235	.0658	.0153	.0040	.0011	.0025	.0010
49,000						.1323	.0716	.0181	.0053	.0013	.0028	.0010
50,000						.1398	.0768	.0224	.0063	.0013	.0031	.0010
51,000						.1493	.0819	.0255	.0085	.0015	.0031	.0011
52,000						0.1580	.0879	.0301	.0108	.0018	.0035	.0013
53,000							.0936	.0351	.0135	.0020	.0040	.0014
54,000							.0989	.0406	.0165	.0021	.0043	.0015
55,000							0.1063	0.0471	0.0210	0.0022	0.0043	0.0016

* Each 1,000 pounds of load as given in Column I = 2,052.7 pounds per square inch = 139.64 atmospheres.

† Between the daggers in the column pertaining to Alabaster the results are irregular because in one of the experiments there was an impact delivered to the specimen, as explained in the discussion of Fig. 12.

TABLE I—*Continued*

Load in Pounds*	Tallow	Lead	Copper	Rocks Employed								
				Steatite, Virginia, U.S.A.	Alabaster, Castelino, Italy	Sandstone, Cleveland, Ohio, U.S.A.	White Marble, Carrara, Italy	Dolomite, Cockeysville, Maryland, U.S.A.	Black Belgian Marble, ("Noir fin")	Slate, New Rockland, Quebec, Canada	Olivine Diabase, Sudbury, Ontario, Canada	Granite, Baveno, Italy
56,000	....	....	....	....	....	....	0.1109	0.0546	0.0258	0.0026	0.0048	0.0016
57,000	....	....	....	....	....	....	.1155	.0604	.0313	.0085	.0051	.0018
58,000	....	....	....	....	....	....	.1219	.0671	.0360	.0091	.0056	.0021
59,000	....	....	....	....	....	....	.1282	.0754	.0413	.0103	.0070	.0023
60,000	....	....	....	....	....	....	.1349	.0818	.0485	.0108	.0078	.0023
61,000	....	....	....	....	....	....	.1405	.0895	.0543	.0119	.0084	.0024
62,000	....	....	....	....	....	....	.1459	.0968	.0613	.0150	.0089	.0026
63,000	....	....	....	....	....	....	0.1505	.1041	.0665	.0335	.0098	.0026
64,000	....	....	....	....	....	....	....	.1110	.0723	.0447	.0113	.0029
65,000	....	....	....	....	....	....	....	.1178	.0838	.0540	.0125	.0031
66,000	....	....	....	....	....	....	....	.1265	.0898	.0680	.0136	.0033
67,000	....	....	....	....	....	....	....	.1344	.0980	.0801	.0159	.0036
68,000	....	....	....	....	....	....	....	.1410	.1073	.0905	.0178	.0036
69,000	....	....	....	....	....	....	....	.1494	.1143	.0981	.0206	.0041
70,000	....	....	....	....	....	....	....	.1569	.1235	.1116	.0247	.0045
71,000	....	....	....	....	....	....	....	0.1651	.1315	.1223	.0289	.0049
72,000	....	....	....	....	....	....	....	....	.1395	.1316	.0330	.0050
73,000	....	....	....	....	....	....	....	....	.1540	.1432	.0396	.0055
74,000	....	....	....	....	....	....	....	....	0.1648	0.1521	.0475	.0060
75,000	....	....	....	....	....	....	....	....	....	....	.0554	.0069
76,000	....	....	....	....	....	....	....	....	....	....	.0650	.0076
77,000	....	....	....	....	....	....	....	....	....	....	.0744	.0089
78,000	....	....	....	....	....	....	....	....	....	....	.0839	.0121
79,000	....	....	....	....	....	....	....	....	....	....	.0939	.0290

80,000	....	....	....	....	....	....	....	....	....	....	.1038	.0434
81,000	....	....	....	....	....	....	....	....	....	....	.1131	.0554
82,000	....	....	....	....	....	....	....	....	....	....	.1230	.0636
83,000	....	....	....	....	....	....	....	....	....	....	.1321	.0708
84,000	....	....	....	....	....	....	....	....	....	....	.1405	.0793
85,000	....	....	....	....	....	....	....	....	....	....	0.1505	.0868
86,000	....	....	....	....	....	....	....	....	....	....	....	.0961
87,000	....	....	....	....	....	....	....	....	....	....	....	.1000
88,000	....	....	....	....	....	....	....	....	....	....	....	.1143
89,000	....	....	....	....	....	....	....	....	....	....	....	.1234
90,000	....	....	....	....	....	....	....	....	....	....	....	.1308
91,000	....	....	....	....	....	....	....	....	....	....	....	.1418
92,000	....	....	....	....	....	....	....	....	....	....	....	0.1498
Number of experiments performed	3	2	2	4	2	2	4	2	2	2	2	2
Average duration of each experiment	0 hr. 52 min.	1 hr. 10 min.	1 hr. 32 min.	2 hr. 11 min.	2 hr. 22 min.	4 hr. 0 min.	4 hr. 0 min.	4 hr. 32 min.	4 hr. 14 min.	3 hr. 49 min.	4 hr. 56 min.	4 hr. 11 min.

*Each 1,000 pounds of load as given in Column I = 2,052.7 pounds per square inch = 139.64 atmospheres.

TABLE II

AMOUNT OF DEFORMATION, IN INCHES, OF THE SEVERAL ROCKS WHEN INCLOSED IN THE STEEL CYLINDERS HAVING A WALL THICKNESS OF 0.33 CENTIMETER

LOAD IN POUNDS*	TALLOW	LEAD	COPPER	ROCKS EMPLOYED									
				Steatite, Virginia, U.S.A.	Alabaster, Castelino, Italy	Sandstone, Cleveland, Ohio, U.S.A.	White Marble, Carrara, Italy	Dolomite, Cockeysville, Maryland, U.S.A.	Black Belgian Marble, ("Noir fin")	Slate, New Rockland, Quebec, Canada	Olivine Diabase, Sudbury, Ontario, Canada	Granite, Baveno, Italy
4,000	....	....	....	....	....	....	....	....	....	....	....	....
5,000	0.0004	....	....	....	....	....	....	....	....	....	....	....
6,000	.0005	0.0005	....	....	....	....	....	....	....	....	....	....
7,000	.0008	.0008	....	....	....	....	....	....	....	....	....	....
8,000	.0010	.0013	....	....	....	....	....	....	....	....	....	....
9,000	.0012	.0013	....	....	....	....	....	....	....	....	....	....
10,000	0018	.0019	0.0003	....	....	....	....	....	....	....	....	....
11,000	.0028	.0025	.0006	0.0004	....	....	....	....	....	....	....	....
12,000	.0055	.0040	.0010	.0005	....	....	....	....	....	....	....	....
13,000	.0095	.0080	.0013	.0007	0.0004	....	....	....	....	....	....	....
14,000	.0153	.0116	.0017	.0009	.0006	....	0.0005	....	....	....	....	....
15,000	.0212	.0173	.0018	.0010	.0006	....	.0008	....	....	....	....	....
16,000	.0280	.0240	.0023	.0013	.0006	0.0003	.0010	....	....	....	....	....
17,000	.0367	.0313	.0026	.0013	.0008	.0003	.0010	....	....	....	....	....
18,000	.0475	.0419	.0033	.0017	.0008	.0005	.0010	....	....	....	....	....
19,000	.0645	.0560	.0040	.0021	.0009	.0005	.0010	....	....	....	....	....
20,000	.0848	0.0805	.0050	.0025	.0009	.0008	.0013	....	....	....	....	....
21,000	0.1152	....	.0065	.0028	.0013	.0008	.0013	....	....	....	....	....
22,000	....	....	.0078	.0029	.0014	.0008	.0015	0.0003	....	....	....	....
23,000	....	....	.0098	.0034	.0015	.0008	.0018	.0003	....	....	....	....
24,000	....	....	.0117	.0038	.0019	.0008	.0020	.0003	....	....	....	....

25,000	....	....	.0138	.0050	.0026	.0008	.0020	.0003	....	....	....	....
26,000	....	....	.0155	.0071	.0029	.0010	.0020	.0003	....	....	....	....
27,000	....	....	.0187	.0093	.0031	.0010	.0023	.0003	....	....	....	....
28,000	....	....	.0212	.0116	.0038	.0010	.0023	.0004	0.0003	....	....	....
29,000	....	....	.0238	.0146	.0043	.0010	.0028	.0006	.0003	....	....	....
30,000	....	....	.0273	.0173	.0051	.0011	.0030	.0008	.0003	....	....	....
31,000	....	....	.0305	.0212	.0061	.0013	.0040	.0009	.0003	....	....	....
32,000	....	....	.0345	.0237	.0075	.0013	.0040	.0010	.0007	....	....	....
33,000	....	....	.0387	.0265	.0089	.0018	.0050	.0011	.0007	....	....	....
34,000	....	....	.0426	.0306	.0114	.0043	.0063	.0011	.0007	....	....	....
35,000	....	....	.0470	.0333	.0126	.0058	.0078	.0013	.0007	....	0.0003	
36,000	....	....	.0518	.0379	.0163	.0084	.0085	.0014	.0007	....	.0003	0.0005
37,000	....	....	.0570	.0415	.0186	.0106	.0093	.0014	.0008	....	.0005	.0005
38,000	....	....	.0630	.0456	.0209	.0141	.0108	.0015	.0009	....	.0009	.0005
39,000	....	....	.0688	.0481	.0238	.0164	.0128	.0018	.0009	....	.0009	.0005
40,000	....	....	.0760	.0537	.0269	.0188	.0140	.0019	.0009	....	.0009	.0005
41,000	....	....	.0838	.0601	.0309	.0210	.0150	.0021	.0010	....	.0009	.0005
42,000	....	....	.0915	.0658	.0348	.0243	.0170	.0021	.0012	0.0007	.0011	.0005
43,000	....	....	0.0987	.0714	.0370	.0273	.0185	.0024	.0013	.0007	.0011	.0005
44,000	....	....	....	.0789	.0451	.0298	.0210	.0026	.0013	.0009	.0011	.0010
45,000	....	....	....	.0856	.0494	.0325	.0230	.0028	.0013	.0009	.0011	.0010
46,000	....	....	....	.0939	.0558	.0349	.0258	.0031	.0013	.0009	.0014	.0010
47,000	....	....	....	.1023	.0615	.0389	.0270	.0040	.0015	.0009	.0014	.0010
48,000	....	....	....	.1117	.0680	.0427	.0300	.0044	.0016	.0009	.0014	.0010
49,000	....	....	....	.1217	.0742	.0452	.0320	.0054	.0017	.0011	.0014	.0010
50,000	....	....	....	.1333	.0820	.0493	.0358	.0063	.0022	.0011	.0014	.0010
51,000	....	....	....	.1428	.0922	.0535	.0373	.0068	.0025	.0013	.0017	.0010
52,000	....	....	....	0.1525	.0980	.0570	.0408	.0086	.0030	.0013	.0017	.0010
53,000	....	....	....	....	.1074	.0610	.0458	.0096	.0042	.0013	.0017	.0010
54,000	....	....	....	....	.1175	.0656	.0485	.0114	.0053	.0013	.0021	.0010
55,000	....	....	....	....	.1265	.0695	.0530	.0131	.0073	.0015	.0021	.0015
56,000	....	....	....	....	.1356	.0740	.0558	.0155	.0084	.0016	.0023	.0015
57,000	....	....	....	....	.1456	.0783	.0600	.0179	.0110	.0018	.0025	.0015
58,000	....	....	....	....	0.1569	0.0830	0.0630	0.0204	0.0138	0.0019	0.0025	0.0015

* Each 1,000 pounds of load as given in Column I = 2,052.7 pounds per square inch = 139.64 atmospheres.

TABLE II—*Continued*

Load in Pounds*	Tallow	Lead	Copper	Rocks Employed								
				Steatite, Virginia, U.S.A.	Alabaster, Castelino, Italy	Sandstone, Cleveland, Ohio, U.S.A.	White Marble, Carrara, Italy	Dolomite, Cockeysville, Maryland, U.S.A.	Black Belgian Marble, ("Noir fin")	Slate, New Rockland, Quebec, Canada	Olivine Diabase, Sudbury, Ontario, Canada	Granite, Baveno, Italy
59,000	....	....	....	....	....	0.0887	0.0658	0.0235	0.0175	0.0019	0.0028	0.0020
60,000	....	....	....	....	....	.0952	.0690	.0256	.0204	.0020	.0030	.0021
61,000	....	....	....	....	....	.1012	.0728	.0300	.0237	.0023	.0030	.0023
62,000	....	....	....	....	....	.1060	.0765	.0336	.0274	.0025	.0034	.0024
63,000	....	....	....	....	....	.1115	.0825	.0370	.0312	.0029	.0038	.0024
64,000	....	....	....	....	....	.1168	.0865	.0410	.0339	.0030	.0040	.0026
65,000	....	....	....	....	....	.1223	.0900	.0450	.0380	.0030	.0043	.0027
66,000	....	....	....	....	....	.1278	.0938	.0496	.0424	.0035	.0048	.0028
67,000	....	....	....	....	....	.1330	.1013	.0544	.0466	.0035	.0050	.0031
68,000	....	....	....	....	....	.1378	.1055	.0588†	.0506	.0039	.0052	.0031
69,000	....	....	....	....	....	.1445	.1103	.0646†	.0550	.0040	.0056	.0034
70,000	....	....	....	....	....	.1498	.1155	.0681	.0585	.0041	.0056	.0035
71,000	....	....	....	....	....	0.1553	.1205	.0719	.0649	.0046	.0066	.0036
72,000	....	....	....	....	....	....	.1253	.0761	.0697	.0050	.0073	.0036
73,000	....	....	....	....	....	....	.1270	.0810	.0760	.0050	.0083	.0037
74,000	....	....	....	....	....	....	.1353	.0858	.0806	.0056	.0090	.0039
75,000	....	....	....	....	....	....	.1400	.0909	.0844	.0060	.0096	.0041
76,000	....	....	....	....	....	....	.1441	.0964	.0900	.0135	.0105	.0043
77,000	....	....	....	....	....	....	.1488	.1025	.0960	.0226	.0115	.0048
78,000	....	....	....	....	....	....	.1540	.1080	.1017	.0273	.0129	.0052
79,000	....	....	....	....	....	....	.1583	.1131	.1059	.0300	.0139	.0055
80,000	....	....	....	....	....	....	.1635	.1174	.1139	.0356	.0149	.0058
81,000	....	....	....	....	....	....	.1685	.1229	.1194	.0525	.0176	.0065
82,000	....	....	....	....	....	....	0.1738	.1290	.1247	.0609	.0199	.0072

83,000	...	...	...	...	...	...	...	.1365	.1298	.0675	.0223	.0079
84,000	...	...	...	...	...	...	...	.1428	.1373	.0748	.0255	.0087
85,000	...	...	...	...	...	...	...	.1476	.1438	.0814	.0288	.0098
86,000	...	...	...	...	...	...	...	0.1543	0.1495	.0894	.0351	.0117
87,000	...	...	...	...	...	...	...	...	...	.0970	.0420	.0136
88,000	...	...	...	...	...	...	...	...	...	.1053	.0484	.0159
89,000	...	...	...	...	...	...	...	...	...	.1120	.0575	.0183
90,000	...	...	...	...	...	...	...	...	...	.1194	.0609	.0213
91,000	...	...	...	...	...	...	...	...	...	.1263	.0678	.0249
92,000	...	...	...	...	...	...	...	...	...	.1320	.0739	.0308
93,000	...	...	...	...	...	...	...	...	...	0.1407	.0808	.0341
94,000	...	...	...	...	...	...	...	...	...	...	.0864	.0402
95,000	...	...	...	...	...	...	...	...	...	...	.0928	.0444
96,000	...	...	...	...	...	...	...	...	...	...	.0987	.0493
97,000	...	...	...	...	...	...	...	...	...	...	.1067	.0548
98,000	...	...	...	...	...	...	...	...	...	...	.1133	0607
99,000	...	...	...	...	...	...	...	...	...	...	.1198	.0651
100,000	...	...	...	...	...	...	...	...	...	...	.1277	.0718
101,000	...	...	...	...	...	...	...	...	...	...	0.1349	.0793
102,000	...	...	...	...	...	...	...	...	...	...	...	.0859
103,000	...	...	...	...	...	...	...	...	...	...	...	.0896
104,000	...	...	...	...	...	...	...	...	...	...	...	.0978
105,000	...	...	...	...	...	...	...	...	...	...	...	.1048
106,000	...	...	...	...	...	...	...	...	...	...	...	.1106
107,000	...	...	...	...	...	...	...	...	...	...	...	.1170
108,000	...	...	...	...	...	...	...	...	...	...	...	0.1231
Number of experiments performed	2	2	3	5	2	2	3	3	3	2	2	4
Average duration of each experiment	1 hr. 4 min.	1 hr. 20 min.	2 hr. 3 min.	3 hr. 1 min.	3 hr. 32 min.	5 hr. 36 min.	6 hr. 0 min.	5 hr. 26 min.	4 hr. 39 min.	4 hr. 23 min.	5 hr. 14 min.	5 hr. 9 min.

*Each 1,000 pounds of load as given in Column I = 2,052.7 pounds per square inch = 139.64 atmospheres.

† Between the daggers in the column pertaining to Dolomite the results are irregular because in one of the experiments the load was removed and when later restored a slight impact was delivered to the deforming rock column.

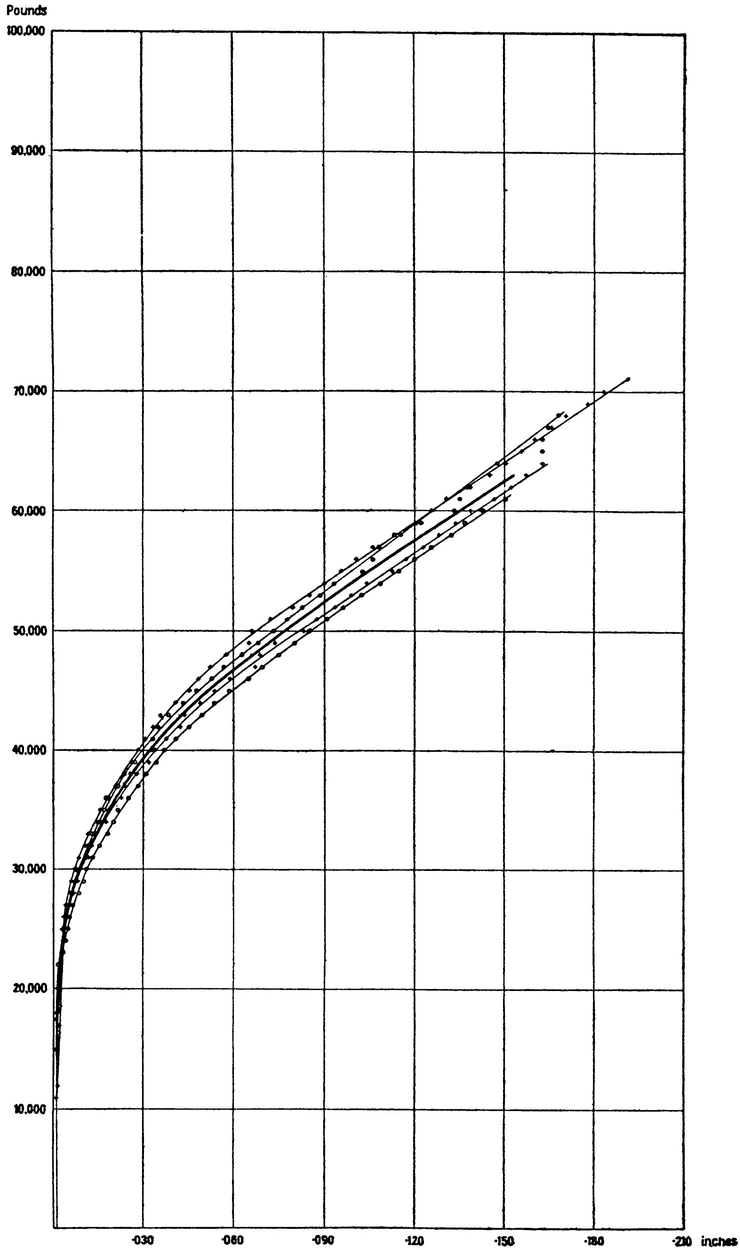

FIG. 5.—Curves showing graphically the results obtained in four experiments on the deformation of Carrara marble when it is inclosed in a steel cylinder with wall 0.25 cm. thick—also the mean of these curves (in heavy line).

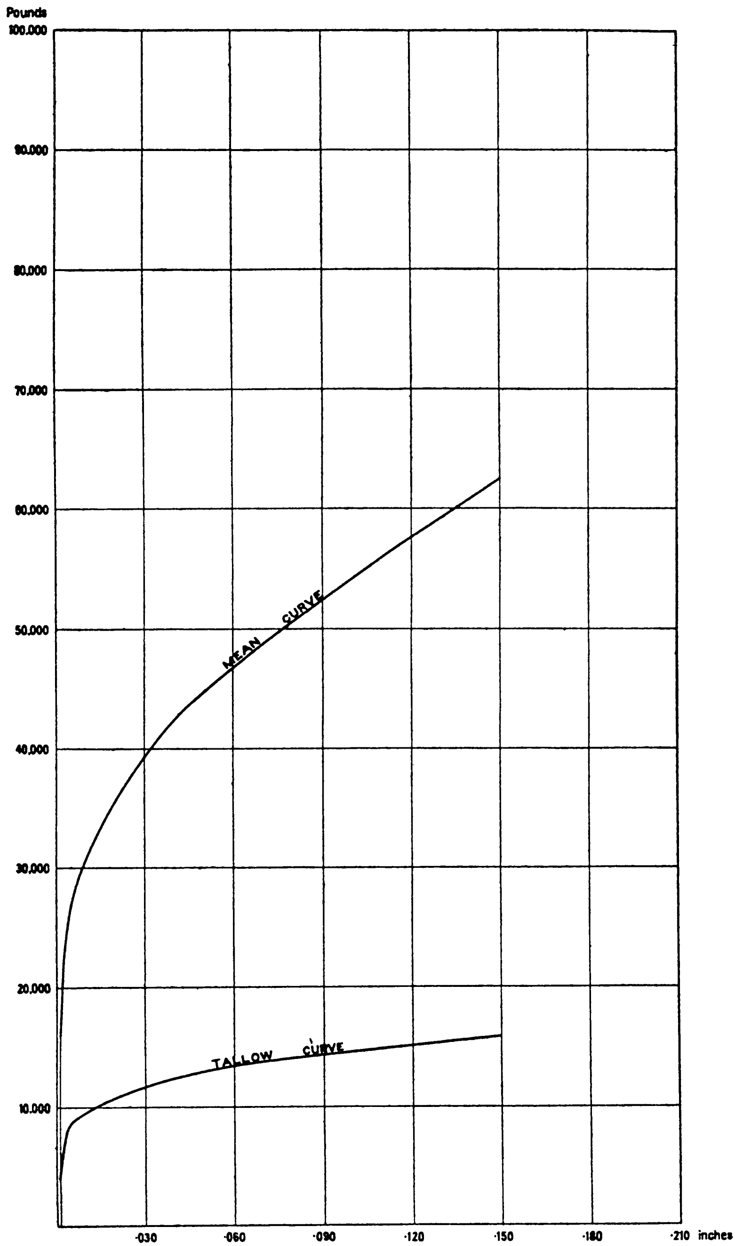

FIG. 6.—The mean of the curves obtained in the deformation of Carrara marble (see Fig. 5) when it is inclosed in a steel cylinder with wall 0.25 cm. thick—with the curve obtained when a steel tube of identical dimensions is deformed when filled with tallow.

Such being the case, with the information thus secured it is possible to separate the two components of the load, namely, that necessary to overcome the resistance offered by the tube and that required to effect the deformation of the marble. If at a series of points the load required to produce a certain distension or bulge in the steel tube when filled with the tallow is subtracted from the load required to produce the same bulge in the case of the tube containing the marble, values are obtained which represent that portion of the load which is expended in affecting the deformation of the marble. This may be termed the *true curve*, and that obtained for a standard column of Carrara marble deformed in a standard steel tube having a wall thickness of 0.25 centimeter is shown in Fig. 7. In the same manner the *true curve* for each of the other rocks may be plotted from the data presented in Tables I and II. It will be seen that, in the case of Carrara marble, this curve starting from a distension of 0.001, which may be considered to be due to elastic deformation, and which is produced by a load of 12,000 pounds, shows a rapid deflection to a point representing a distension of 0.052 which is produced by a load of 33,000 pounds, after which it develops into what is practically a straight line until the tube ruptures.

This shows that after the elastic limit of the marble has been passed, at about 12,000 pounds, and the marble commences to deform, the load which is required to start this movement and produce a unit of diametral expansion is relatively great. As the movement progresses the additional increment of load required to produce a unit of diametral expansion grows progressively less till a bulge of 0.052 is reached, after which there is a definite and constant ratio between the increase of load and the expansion which it produces. This ratio is 0.0065 for each increase of 1,000 pounds in load.

It will be noted that in the case of the slate, just after the rock began to deform, the curve shows a sudden break or sag which is repeated at a second point before the regular movement, indicated by the nearly straight line, is developed. This is due to the fact, above mentioned, that the slate, being a foliated and not a granular rock, is not isotropic in its response to pressure. It consists of little

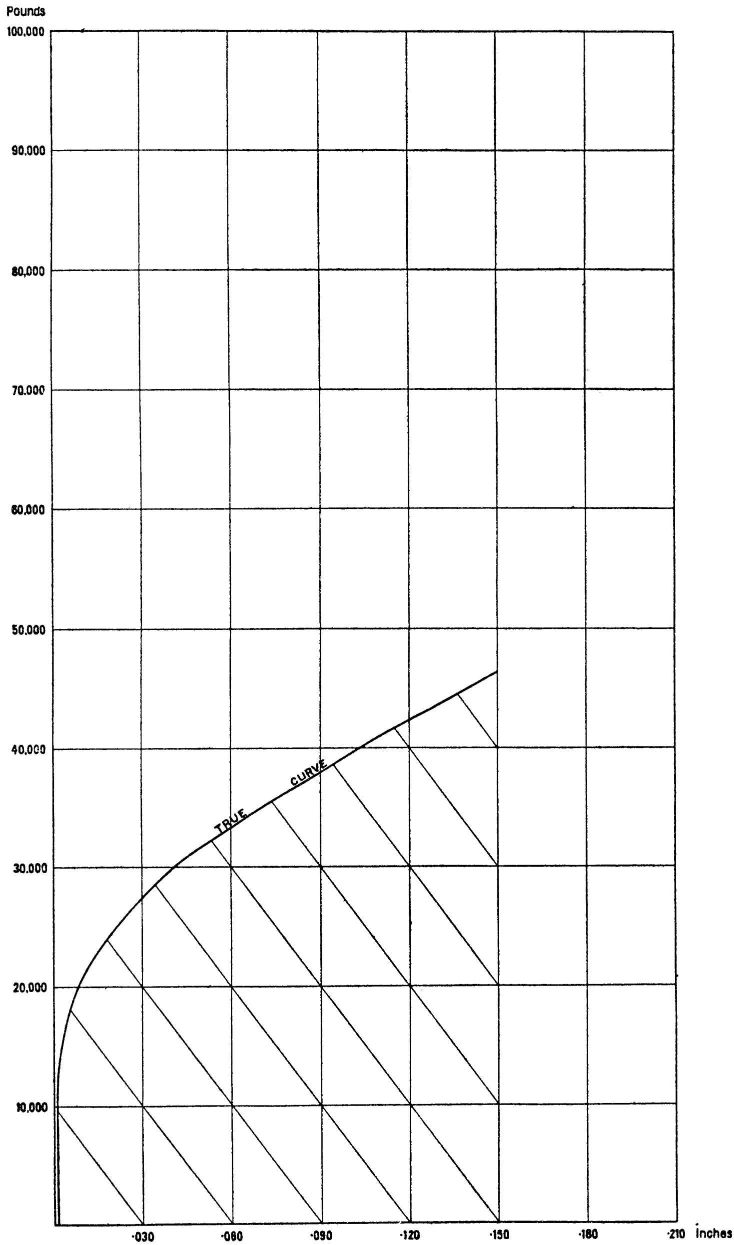

FIG. 7.—True curve obtained by the deformation of a standard column of Carrara marble in steel cylinder with wall 0.25 cm. thick. The area designated by oblique lines represents the work done in effecting the deformation of the marble to a bulge of 0.150 inch.

plates of kaolin and muscovite lying parallel to one another and at right angles to the direction in which the pressure is exerted. The breaking down of the foliated structure of the rock is indicated on the curves by the irregularities to which reference has been made.

It will also be seen that in the case of granite, when the lateral resistance is relatively low (e.g., when the rock is inclosed in the steel tube having a 0.25-centimeter wall), there is at the same point a sag, though much less marked, due to the fact that the lateral resistance offered by the tube is not quite sufficient to develop a uniform movement in this the strongest of all the rocks employed in the investigation.

Attention must be drawn to the manner in which deformation goes forward in a column of rock when deformed under the conditions of the experiment. As may be seen, if the tube and the inclosed rock are sawed in two vertically, the column of rock begins to move or flow at the middle, the motion taking place first along the well-known shearing cones, having an angle of approximately 45° (usually somewhat greater), seen when a column or cube of the rock is crushed between the faces of a testing machine in the ordinary determinations of the strength of rock for building purposes. Thus, as the movement progresses, there develops within the column two obtuse cones, having as their bases the faces of the advancing pistons and consisting of portions of the rock which show no evidences whatsoever of deformation, but which are, under the conditions of the experiment, subjected only to cubic compression. As the experiment progresses, these cones (see *A* and *B* in Fig. 8) advance into the deforming rock, additional amounts of the rock shearing off the surfaces of the cones and thus coming to participate in the movements which are going forward. Owing to the fact, therefore, that the quantity of flowing rock is continually increasing in an unknown ratio, it is impossible from the data mentioned above to determine whether the definite increase in the ratio of load to deformation is due to an increase of internal friction developed with increase of pressure, or to the increased amount of material which is being moved.

The answer to this question is obtained from another series of experiments which exactly duplicated those with the columns of

Carrara marble, described above, except that the lateral resistance to movement was increased by increasing the thickness of the walls of the steel tube inclosing the marble from a thickness of 0.25 centimeter to 0.33 centimeter. In these the amount of material moved is identical with that in the series of experiments just described, while the internal friction is increased by the increased thickness of the steel tube.

A series of additional experiments were also made to determine the resistance offered by such tubes when filled with soft tallow.

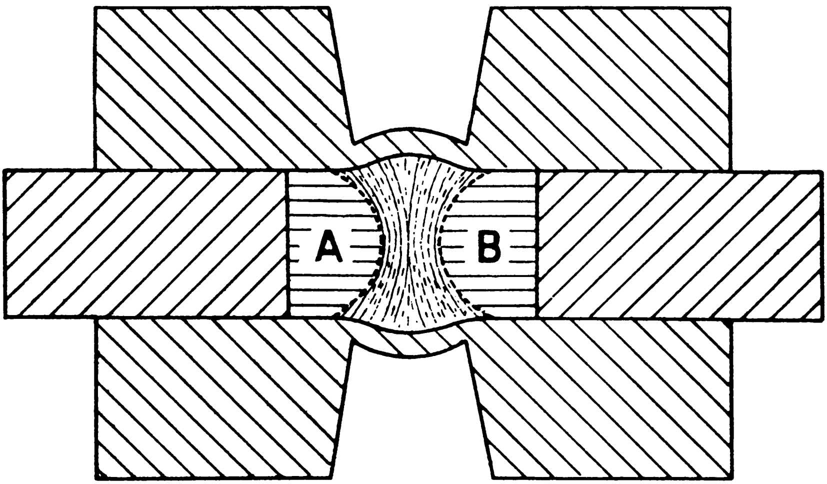

FIG. 8.—Longitudinal section through steel cylinder with pistons inserted and inclosing a deformed column of rock—showing the obtuse shearing cones which advance into the deforming rock.

In this way another series of curves were obtained for each material and another "true curve" for the deformation of a standard column of Carrara marble under conditions identical with those of the former experiments, except that the resistance to deformation offered by the steel tube was much greater. The "true curve" for the deformation of the marble in a steel tube having walls 0.33 centimeter thick is shown in Fig. 10.

An inspection of this curve will show that while, as before, starting from the limit of elastic expansion the rising load at first induces a relatively small amount of movement in the rock, the

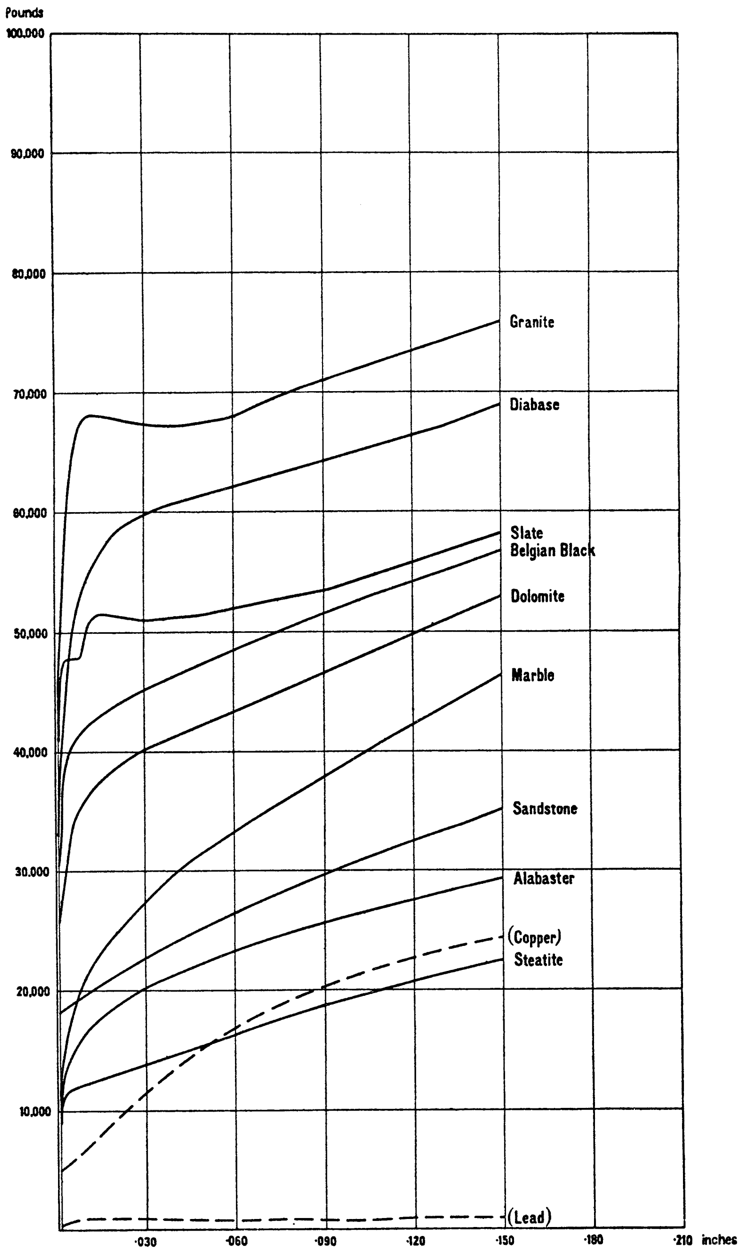

FIG. 9.—True curves obtained by the deformation of the several rocks when inclosed in the steel cylinders with wall 0.25 cm. thick.

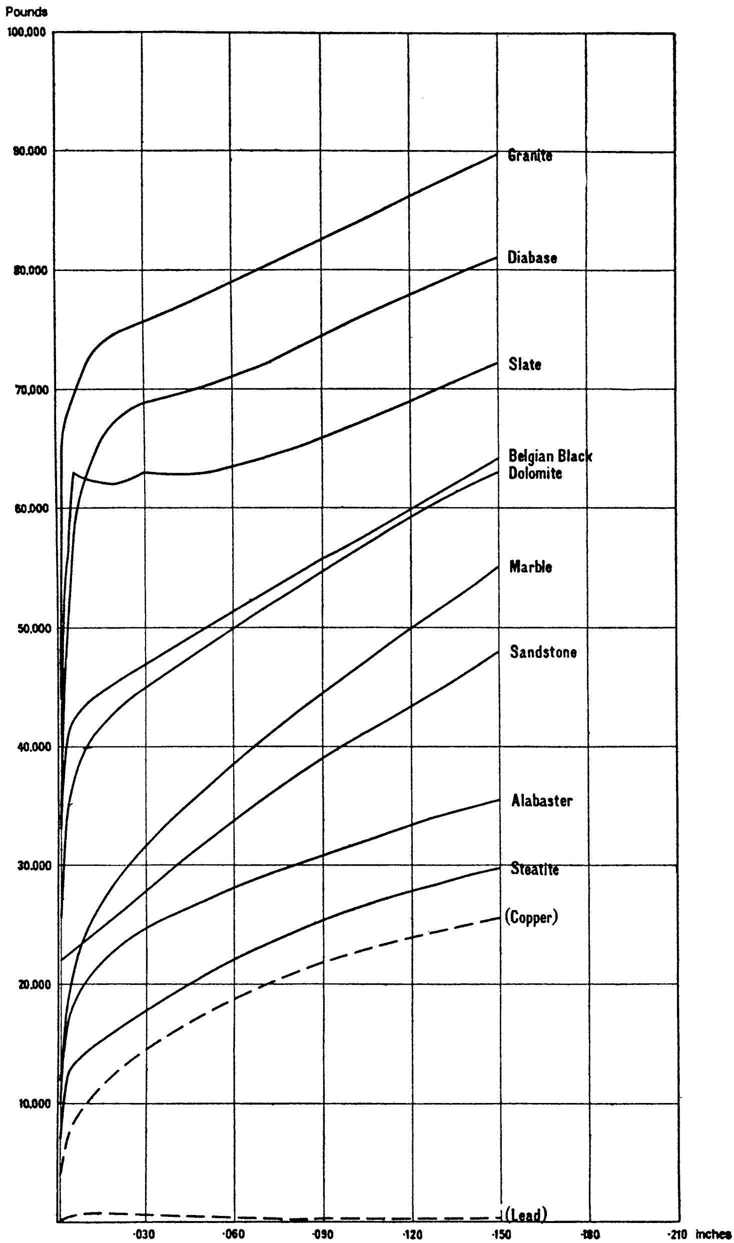

FIG. 10.—True curves obtained by the deformation of the several rocks when inclosed in the steel cylinders with wall 0.33 cm. thick.

ratio of the amount of this movement to increment of loads increases rather rapidly, and, after deformation amounting to about 0.06 has been brought about—which requires a load of 38,750 pounds—the ratio of increase of load to amount of deformation of the column becomes constant, as when the marble is deformed in the tubes with thinner walls. It will be seen, however, that for the experiments in the thicker-walled tube this ratio of increase is much less than when the wall was thinner, i.e., 0.25 centimeter being 0.0051 diametrical increase for each increase of 1,000 pounds in the load, instead of .0065, as in the first series of experiments.

This demonstrates that the moving rock possesses internal friction and that with the increase of the lateral resistance the amount or coefficient of friction rapidly increases, and at a constant ratio.

The investigation was then extended to the other rocks of the series enumerated on pp. 598 and 599. The conditions and method of conducting the experiments were in every case identical with those just described with Carrara marble. Two sets of standard steel tubes, having wall thicknesses of 0.25 centimeter and 0.33 centimeter, respectively, were employed, and the true curves were plotted representing the mean of a series of experiments in each case (see Figs. 9 and 10).

"WORK DONE" IN THE DEFORMATION OF ROCKS

If Px be the load to which the specimen is subjected and Py be the resistance to movement offered by the inclosing walls of the steel cylinder, the data were first examined to ascertain whether the formula

$$Px - Py = \text{a constant}$$

represents the movement, and it was found that this was not the case. They were then studied to see whether each rock possessed a constant factor K, which might be termed its modulus of plasticity; as in the formula

$$Px - KPy = \text{a constant}$$

It was found that, if the data are calculated so as to take into consideration the bulge of the cylinder and are plotted to show

vertical stress as compared with lateral stress, this formula represents the facts and that each of the softer rocks possesses a definite modulus of plasticity, this being also true in the harder rocks in the earlier stages of the deformation at least.

This interesting fact is discussed at length in the accompanying paper by Dr. King, where a mathematical treatment of some of the new data developed in the present investigation is also presented, illuminating certain parts at least of that hitherto unsubdued and almost unoccupied domain—the mathematics of the flow of solids.

In the present paper, without entering into a mathematical treatment of the subject, the following deductions from the experimental data may be indicated.

If a vertical line be drawn cutting off the "true curve" obtained in the case of any rock when the deformation of the tube amounting to 0.15 has been reached, and if the area inclosed by this line, the "true curve" itself, and the base line of the diagram be measured, this area represents the "work done" to effect the deformation of the rock. This area showing the "work done" in deforming a standard column of Carrara marble in a 0.25-centimeter steel tube in Fig. 7 is shaded. In Fig. 9 the "true curves" obtained in this deformation of all the rocks of the series, in steel tubes having a wall thickness of 0.25 centimeter, are shown, and in Fig. 10 the complete series of "true curves" obtained when the wall thickness of the tube is increased to 0.33 centimeter is set forth. In both figures the curves are cut off at the ordinate 0.15, and the area representing the "work done" in the case of each rock is clearly shown and may be compared.

Table III sets forth these comparative values in square inches. This table shows quite clearly that with the increased resistance, offered by the thicker-walled steel tube, the amount of work required to effect an equal deformation increased in the case of every rock. It also sets forth the comparative value of these increases and also the relative amount of work done to deform the different rocks of the series.

The table thus shows that the "work done" in deforming a column of marble of the size employed and under the conditions of the experiment, when inclosed in the thinner-walled tube, is to the "work done" when an identical column is deformed, when inclosed

in the thicker-walled tube, as 51,708 is to 60,415. Or, again, that the "work done" in deforming a marble column, whether the resistance be small or great, is almost exactly one-half of that required to effect an equal amount of deformation in a column of granite under the same conditions. That is to say, almost exactly twice as much work is required to deform granite as is required to effect an equal deformation in the case of marble and nearly four times as much as is required to produce an equal deformation in the case of steatite.

TABLE III

RELATIVE AMOUNT OF "WORK DONE" IN EFFECTING AN EQUAL DEFORMATION IN UNIT COLUMNS OF DIFFERENT ROCKS

	Under Resistance of	
	0.25 cm. Steel Tube	0.33 cm. Steel Tube
Steatite	26,054	34,123
Alabaster	35,569	42,946
Sandstone	41,262	53,446
Marble	51,708	60,415
Dolomite	66,362	77,092
Belgian Black	73,754	79,362
Slate	79,069	97,154
Diabase	92,985	107,431
Granite	104,169	119,877

TABLE IV

RELATIVE AMOUNT OF "WORK DONE" IN EFFECTING AN EQUAL DEFORMATION IN UNIT COLUMNS OF DIFFERENT ROCKS CALCULATED ON THE BASIS OF MARBLE AS UNITY

	Under Resistance of	
	0.25 cm. Steel Tube	0.33 cm. Steel Tube
Steatite	0.50	0.56
Alabaster	0.69	0.71
Sandstone	0.80	0.88
Marble	1.00	1.00
Dolomite	1.28	1.28
Belgian Black	1.43	1.31
Slate	1.53	1.61
Diabase	1.80	1.78
Granite	2.01	1.98

If the "work done" to deform marble be taken as unity, these figures may be set forth as in Table IV.

In these tables there is expressed in actual values the phenomena which are displayed in such a striking manner in the great exposures of the Grenville series and in other terranes which have undergone deformation at great depths below the surface of the earth where the same force has acted on a complex of rocks of diverse character. In these occurrences some of these rocks are torn to fragments, which are then carried far apart in a flowing matrix formed of some other and more plastic member of the complex. This is seen in a striking manner where dykes of diabase or belts of granite cut through a limestone, and the whole complex is then deformed under conditions of deep-seated differential pressure. The diabase dyke or belt of granite is torn apart into angular fragments, which are floated along in sinuous curves in the plastic flowing limestone, like logs or drifting timber on the surface of a flowing river (see Fig. 11).

EFFECT OF A CHANGE IN THE RAPIDITY OF THE APPLICATION OF PRESSURE

In Fig. 12 there are two curves: one showing the deformation of alabaster, the other, the deformation of marble. These also illustrate the effects of a change in the rate at which the pressure is applied.

In the former case, after a load of 36,000 pounds had been gradually applied in successive increments and no movement had taken place under the load for 2 minutes, the next increment of load was by mistake applied suddenly, thereby submitting the rock to an impact instead of to a slow increase of pressure. This, as will be seen, produced at once a movement of 0.045 inch. Following this, however, four increments of load, each of 1,000 pounds, had to be applied before the movement was resumed, and two additional increments, each of 1,000 pounds, had to be applied before the movement could be re-established in its regular course, after which the flow continued in the line followed by the normal curve.

In the second case—that of the marble—the normal course of the experiment was interrupted four times by postponing the time of reading the deformation produced by a new increment of load much longer than usual, namely, from 9 to 75 minutes. These were when the load on the column of rock was 40,000, 55,000,

60,000, and 65,000 pounds, respectively. It will be noted that the same effect, though on a smaller scale, was produced as that just described as the result of impact. An abnormal increase of load

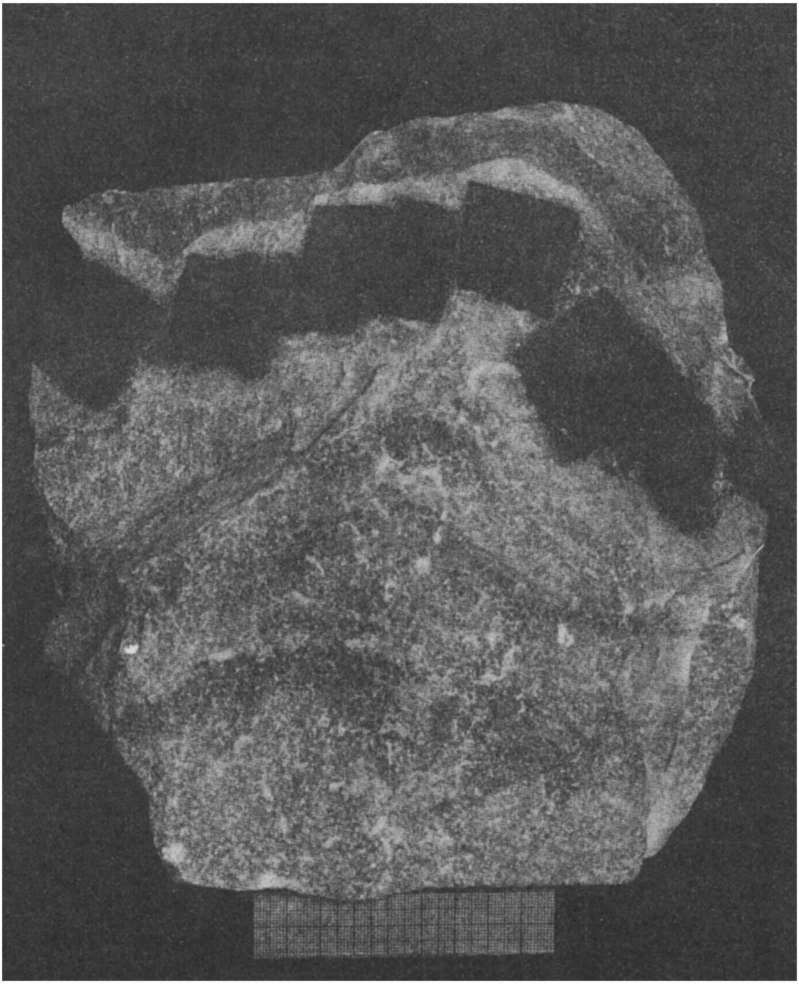

FIG. 11.—Photograph of a specimen of Trenton limestone which has been cut by a narrow dyke of camptonite. The whole has then been distorted by pressure exerted by the intrusion of the igneous mass constituting Mount Royal. The harder camptonite has been broken into fragments which have been carried apart in the flowing mass of more plastic limestone (Canadian Northern Railway Tunnel through Mount Royal, Montreal, Canada).

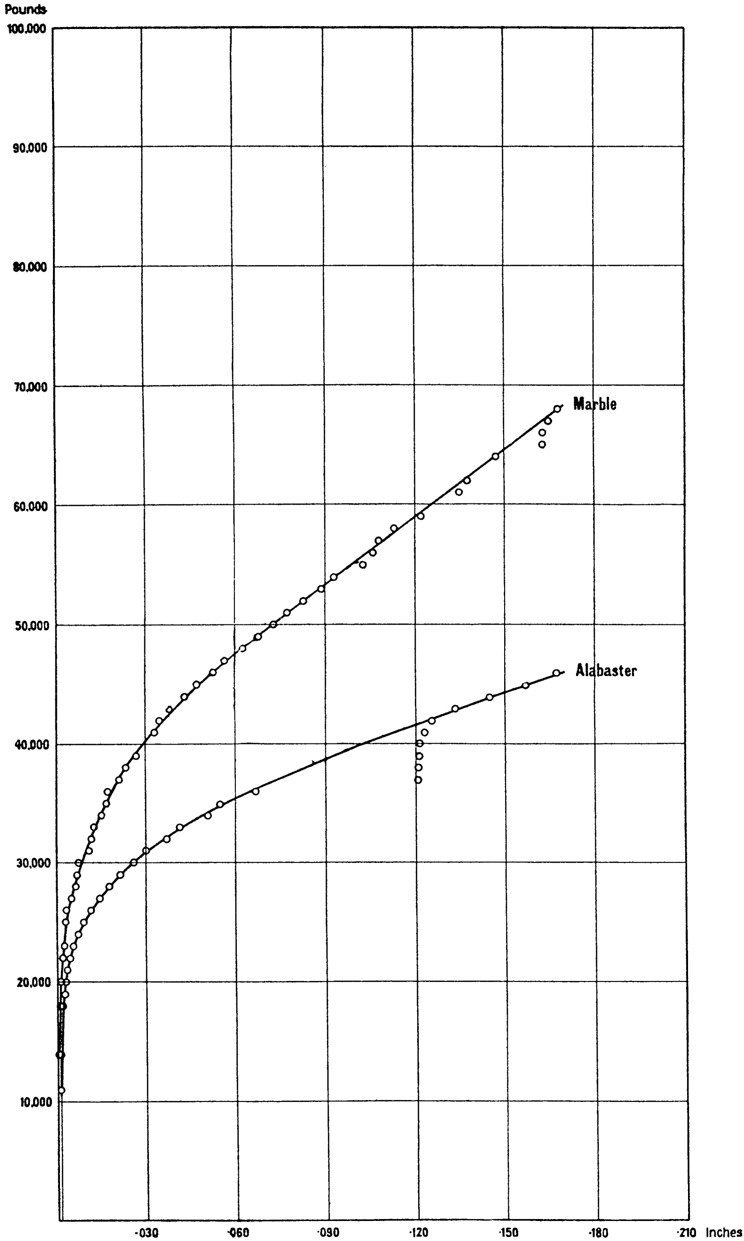

FIG. 12.—Curves showing the effect of change in rate of application of pressure

was required to bring about a re-establishment of the movement, which, however, eventually resumed its former course.

BEARING OF THE RESULTS ON CERTAIN PROBLEMS PRESENTED BY THE EARTH'S CRUST

The experimental results afford a reply to the question propounded by Dr. Gilbert and set forth in the opening paragraph of this paper. They also have a direct bearing on the problems presented by the origin of "decken" and by the theory of isostasy.

When movement producing deformation is once started in the rock under the influence of tangential thrust, resulting in the breaking down of its texture, the rock, if deeply buried in the earth's crust, does not on that account offer a decreased resistance to further movement.

Some experiments by Karman[1] on the deformation of marble under differential pressure have yielded data with reference to the amount of this pressure which must be exerted in the case of marble in order to induce plastic flow in the rock. The data obtained represent maximum results, because in the experiments the pressure was applied rapidly as compared with that which would be developed in any earth movements, and, also, the factor of heat was not taken into account. It must be noted, however, that heat and a very slow application of the deforming force would produce movements under lower pressures than those made use of in the experimental work. Karman found that, if a column of marble were submitted to a supporting or containing pressure, such as that exerted by the steel tube in our experiments, amounting to 685 atmospheres—which would be equivalent to that exerted by the overlying strata at a depth of 2.53 miles below the surface[2]—it would flow uniformly and continuously under a load of 2,870 atmospheres applied to the ends of the column. If the containing pressure fell below the value mentioned, that is, if the rock occupied a position in the earth's crust nearer the surface, it would speedily crumble and break to pieces, presenting in this way a failure similar

[1] "Festigkeits Versuche unter allseitigem Druck," *Zeit. des Ver. deut. Ingenieure*, October 21, 1911.

[2] F. D. Adams, "Depth of the Zone of Flow in the Earth's Crust," *Journal of Geology*, February, 1912.

to that which is obtained in testing building stones in the laboratory. On the other hand, if the containing or supporting pressure is increased, the load required to produce deformation rapidly increases also, and the experiments seem to indicate that with a containing pressure of about 10,000 atmospheres, which would be equivalent to a depth ot about 22 miles below the surface, it would be impossible to make the marble flow, except under a pressure which would be simply colossal.

Since with the increase of resistance to tangential thrust, that is, with increasing depth below the surface of the earth, the amount of such thrust required to produce movements in the earth's crust increases rapidly, it is evident that the great movements of adjustment by rock flow or transference of material in the earth's crust from one point to another—other than the transference of rock in a molten condition—must take place comparatively near the surface. That is, beneath the zone of fracture where adjustment takes place by faults and overthrusts—in the zone of flow—movements so far as they are determined by pressure are effected with an ease which increases rapidly in proportion to their nearness to the surface.

It would seem, therefore, that it is in the upper part of the zone of flow only that the great "decken," as, for instance, those which are developed in the Alps, are produced. This explains the fact that in the mountain range in question it is the upper "decken" which have moved more rapidly and have extended farther than the lower "decken," where the rock is under the increased load and is consequently much less plastic.

Since with the increase of depth there is a rapid increase in rigidity of the rocks of the earth's crust, it is not difficult to understand how it is that, while great movements may take place near the surface of the earth in the upper part of the zone of flow, the globe itself is "more rigid than steel or glass."

The experimental work also affords at least a first approximation to the determination of the dimensions of the forces which are required in order to effect deformation in the earth's crust in the case at least of the chief types of rocks which make up the crust in question.

In these measurements it must again be noted that the factor of pressure alone was considered, no account being taken of the element of heat in the crust, which would undoubtedly tend to increase the ease of movement.

In the experiments it has been shown, as mentioned, that the resistance to deformation exerted by the wall of the steel tube gradually increases as the experiment progresses. If, however, the value of the resistance is taken at a point where the regular column shows a diametral increase of 0.05 inch (or 6.35 per cent), i.e., when the deformation is well under way and after which it becomes proportional to the increased tangential pressure, this resistance, in the case of the experiment with the steel wall 0.25 centimeter thick, would be equivalent to 26,685 pounds to the square inch, or 1,815 atmospheres, that is, to a depth of 4.2 miles below the surface.

In the case of our experiment with a steel wall 0.33 centimeter thick it would be equivalent to 37,359 pounds per square inch, or 2,542 atmospheres, that is, to a depth of 5.8 miles below the surface.

Thus at these respective depths the additional tangential thrust required to induce a pronounced movement in the case of marble and granite, respectively, would be as shown in Table V.

TABLE V

	At Depth of 4.2 Miles		At Depth of 5.8 Miles	
	Pounds per Square Inch	Atmospheres	Pounds per Square Inch	Atmospheres
Marble...........	66,400	4,517	74,500	5,068
Granite..........	138,500	9,422	159,600	10,857

CONCLUSIONS

1. All the rocks employed in the present investigation can be deformed under differential pressure at ordinary temperatures.

2. In order to effect an equal deformation, it is necessary to employ differential pressures having different values in the case of the several rocks.

3. The ease with which these rocks are deformed has as one of its functions the hardness of the rock (or of the minerals composing it).

4. In the case of the softer rocks—alabaster, steatite, marble, etc.—the deformation is produced by movements due to a slipping within the constituent crystals of the rock on their gliding planes, often accompanied by twinning, the movement in this case being similar to that seen in metals when they are deformed. In the harder rocks the deformation is accompanied by granulation, the texture developed being similar to that found in mylonite.

5. Each of the softer rocks at least has a well-defined modulus of plasticity.

6. The "work done" when a rock is deformed by a tangential thrust, within the earth's crust, increases rapidly with the weight of the superincumbent strata, i.e., with its depth below the surface.

7. The relative ease with which the several rocks will flow under differential pressure is shown in Tables III and IV, which give mathematical expression of the "work done" in deforming standard columns of each rock.

8. A uniform thrust exerted on a prism of the earth's crust may deform and fold the upper portion of the mass, while it will be quite insufficient to produce any movement in the lower part of the same mass.

9. The thrust required to develop deformation, taking no cognizance of the influence of heat or the time effect which might result if the pressure were applied with extreme slowness, in the case of marble, and of granite, is shown by the values given in Table V.

10. To revert to the question propounded by Dr. Gilbert, in order to develop flow in any rock within the earth's crust the rock must be submitted to a differential stress which is greater than that which is required merely to break down its texture and very much greater than that which is sufficient to crush it to pieces under the ordinary conditions which obtain at the surface of the earth.